

# Mathematica 11.3 Integration Test Results

Test results for the 24 problems in "4.6.3.1  $(a+b \csc)^m (d \csc)^n (A+B \csc).m$ "

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \csc(c + dx) (a + a \csc(c + dx)) (A + A \csc(c + dx)) \, dx$$

Optimal (type 3, 51 leaves, 6 steps):

$$-\frac{3 a A \operatorname{ArcTanh}[\cos(c + dx)]}{2 d} - \frac{2 a A \cot(c + dx)}{d} - \frac{a A \cot(c + dx) \csc(c + dx)}{2 d}$$

Result (type 3, 137 leaves):

$$-\frac{2 a A \cot(c + dx)}{d} - \frac{a A \csc\left(\frac{1}{2}(c + dx)\right)^2}{8 d} - \frac{a A \log[\cos\left(\frac{c}{2} + \frac{dx}{2}\right)]}{d} - \frac{a A \log[\cos\left(\frac{1}{2}(c + dx)\right)]}{2 d} + \\ \frac{a A \log[\sin\left(\frac{c}{2} + \frac{dx}{2}\right)]}{d} + \frac{a A \log[\sin\left(\frac{1}{2}(c + dx)\right)]}{2 d} + \frac{a A \sec\left(\frac{1}{2}(c + dx)\right)^2}{8 d}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int (a + a \csc(c + dx)) (A + A \csc(c + dx)) \sin(c + dx) \, dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$2 a A x - \frac{a A \operatorname{ArcTanh}[\cos(c + dx)]}{d} - \frac{a A \cos(c + dx)}{d}$$

Result (type 3, 72 leaves):

$$2 a A x - \frac{a A \cos(c) \cos(dx)}{d} - \frac{a A \log[\cos\left(\frac{c}{2} + \frac{dx}{2}\right)]}{d} + \frac{a A \log[\sin\left(\frac{c}{2} + \frac{dx}{2}\right)]}{d} + \frac{a A \sin(c) \sin(dx)}{d}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \csc(c + dx) (a - a \csc(c + dx)) (A + A \csc(c + dx)) \, dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$-\frac{a A \operatorname{ArcTanh}[\cos(c + dx)]}{2 d} + \frac{a A \cot(c + dx) \csc(c + dx)}{2 d}$$

Result (type 3, 79 leaves):

$$-\frac{a A}{8 d} \left( \frac{\csc^2\left(\frac{1}{2}(c+d x)\right)}{8 d} + \frac{\operatorname{Log}[\cos\left(\frac{1}{2}(c+d x)\right)]}{2 d} - \frac{\operatorname{Log}[\sin\left(\frac{1}{2}(c+d x)\right)]}{2 d} + \frac{\sec^2\left(\frac{1}{2}(c+d x)\right)}{8 d} \right)$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \csc(c+d x) (a+a \csc(c+d x)) (A-a \csc(c+d x)) dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$-\frac{a A \operatorname{ArcTanh}[\cos(c+d x)]}{2 d} + \frac{a A \cot(c+d x) \csc(c+d x)}{2 d}$$

Result (type 3, 79 leaves):

$$-\frac{a A}{8 d} \left( \frac{\csc^2\left(\frac{1}{2}(c+d x)\right)}{8 d} + \frac{\operatorname{Log}[\cos\left(\frac{1}{2}(c+d x)\right)]}{2 d} - \frac{\operatorname{Log}[\sin\left(\frac{1}{2}(c+d x)\right)]}{2 d} + \frac{\sec^2\left(\frac{1}{2}(c+d x)\right)}{8 d} \right)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \csc(c+d x) (a-a \csc(c+d x)) (A-a \csc(c+d x)) dx$$

Optimal (type 3, 51 leaves, 6 steps):

$$-\frac{3 a A \operatorname{ArcTanh}[\cos(c+d x)]}{2 d} + \frac{2 a A \cot(c+d x)}{d} - \frac{a A \cot(c+d x) \csc(c+d x)}{2 d}$$

Result (type 3, 137 leaves):

$$\begin{aligned} & \frac{2 a A \cot(c+d x)}{d} - \frac{a A \csc^2\left(\frac{1}{2}(c+d x)\right)}{8 d} - \frac{a A \operatorname{Log}[\cos\left(\frac{c}{2}+\frac{d x}{2}\right)]}{d} - \frac{a A \operatorname{Log}[\cos\left(\frac{1}{2}(c+d x)\right)]}{2 d} + \\ & \frac{a A \operatorname{Log}[\sin\left(\frac{c}{2}+\frac{d x}{2}\right)]}{d} + \frac{a A \operatorname{Log}[\sin\left(\frac{1}{2}(c+d x)\right)]}{2 d} + \frac{a A \sec^2\left(\frac{1}{2}(c+d x)\right)}{8 d} \end{aligned}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int (a-a \csc(c+d x)) (A-a \csc(c+d x)) \sin(c+d x) dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$-2 a A x - \frac{a A \operatorname{ArcTanh}[\cos(c+d x)]}{d} - \frac{a A \cos(c+d x)}{d}$$

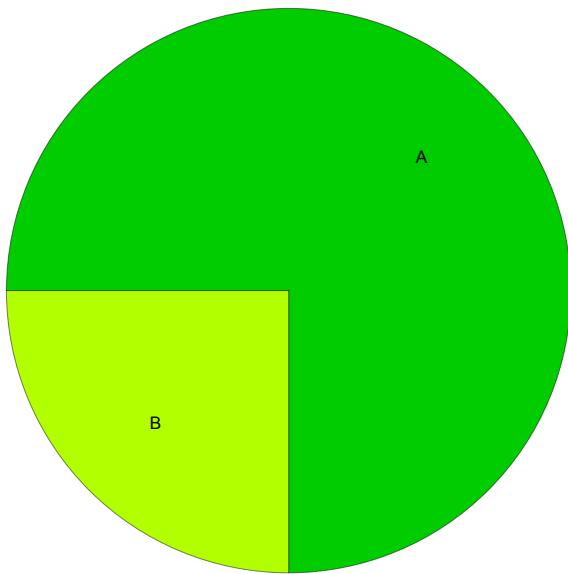
Result (type 3, 72 leaves):

$$-2 a A x - \frac{a A \cos(c) \cos(d x)}{d} - \frac{a A \operatorname{Log}[\cos\left(\frac{c}{2}+\frac{d x}{2}\right)]}{d} + \frac{a A \operatorname{Log}[\sin\left(\frac{c}{2}+\frac{d x}{2}\right)]}{d} + \frac{a A \sin(c) \sin(d x)}{d}$$

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## Summary of Integration Test Results

24 integration problems



A - 18 optimal antiderivatives

B - 6 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts